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A NOTE ON ELECTROLYTIC ANALOGUE EXPERIMENTS FOR THERMAL CONTACT RESISTANCE

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INTRODUCTION

To MODEL the conduction of heat from one solid body to another in a vacuum, the electrical analogue has been used by several workers, [1–4], usually in the form of a cylindrical electrolytic tank intended to represent the flow near a single idealized contact, as shown in Fig. 1. If heat flows at rate $\dot{q}$ in a generally normal direction across a plane interface of apparent area A_a, constriction of the flow into the actual areas of contact causes a complicated three dimensional flow pattern near the interface. The constriction can be regarded as introducing a local fall in temperature ΔT between extrapolated linear temperature distributions, as shown for the single idealized contact in Fig. 1. A thermal contact conductance h_c is generally defined as $(\dot{q}/A_a)/\Delta T$, but in the cylindrical geometry of Fig. 1 it is convenient to express the potential step as a dimensionless resistance given by the ratio of the equivalent length of unconstricted cylinder (l_c) to the radius of the cylinder (r_t). It can be shown [5] that the equations and boundary conditions for the flow are expressible in a dimensionless form which proves that (l_c/r_t) is a function of the shape of contacts alone. Thermal conduction between solids having different thermal conductivities k_1, k_2 can be modelled using an electrolyte tank with a single uniform electrolyte, provided the layout of the solids and the points of supply of heat have mirror symmetry about the interface, because the temperature at the interface is then uniform as shown in [6].

If the idealized contact is a circle of radius r_h on the axis of the cylinder, the solution has been determined numerically by Clausing [7] (the result is quoted in [8]) and can be written in the alternative forms

$$\frac{h_c}{k} = 2\frac{r_h}{A_a}\frac{1}{\psi(r_h/r_t)} \quad \left(\frac{1}{k} = \frac{1}{2}\left\{\frac{1}{k_1} + \frac{1}{k_2}\right\}\right)$$

or

$$\frac{l_c}{r_t} = \frac{\pi}{2}\frac{r_t}{r_h}\psi(r_h/r_t)$$

where the function $\psi(r_h/r_t)$ can be deduced from [7] and is shown in [5] to be closely approximated by $(1 - r_h/r_t)^{1\cdot5}$.

At an actual interface between bodies, it is necessary to allow for many contacts, of various sizes, irregularly spaced across the apparent area of contact. The concepts of "appropriate" and "inappropriate" distribution of contacts have been developed in [6]. Broadly, contacts are "appropriately" distributed if the apparent area of contact can be subdivided so as to define an area around each contact proportional to the heat flow which should occur through it if ΔT is to be the same for all contacts. For such a distribution it is shown in [6] that each contact can be usefully represented by an idealised contact as shown in Fig. 1. If the

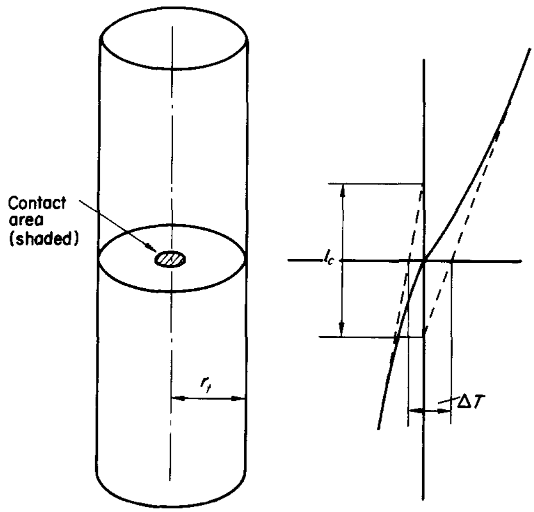


FIG. 1. Idealized single contact.

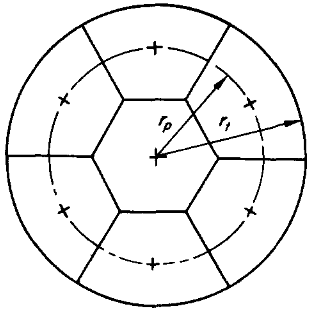
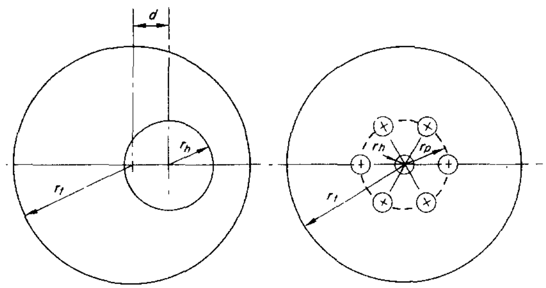


FIG. 2. "Appropriate" distribution of seven equal contact spots, in circular region.

contacts are not so distributed, the resistance at the interface is larger than if the same contacts were appropriately distributed. The "appropriate" distribution for seven equal holes was estimated by dividing the circle of radius r_t into seven equal areas as shown in Fig. 2, and placing the holes at the centroids of those areas. That gave a pitch radius $r_p = 0{\cdot}704\, r_t$.

Several attempts have been made to determine the effects of "inappropriate" distribution, or grouping, of contacts. In one approach, each contact is regarded as being placed eccentrically in a more-or-less cylindrical region, leading to an idealized model with contacts distributed as shown in Fig. 3(a). In a second approach, originating from Holm [9] and developed by Greenwood [10], the resistance of



3(a) 3(b)

FIG. 3. Idealized distributions of contacts, as studied in these experiments. (a) single hole, on or off axis, (b) seven holes, symmetrically placed.

contacts between infinite bodies is regarded as the sum of resistance due to macroconstriction and microconstriction in series. Resistance due to macroconstriction is that for a single contact of radius g, where g is a "group radius" or "Holm radius", and resistance due to microconstriction is that for the actual contacts acting independently of each other in parallel. The problem is then a matter of finding g, and Greenwood [10] considers some calculated cases and deduces that "A general rule of thumb for an irregular cluster is to draw an envelope lying outside each peripheral spot by a distance equal to the centre to centre separation from its nearest neighbour: the Holm radius is approximately that of the circle containing the same area". As discussed in [5] this can be extended to the present case of resistance of contacts between cylindrical bodies if we regard that resistance as the sum of (i) resistance due to macroconstriction, taken as that for a single contact of radius g in the cylinder, where g is determined in accordance with Greenwood's rule of thumb, and (ii) resistance due to microconstriction, taken as the resistance which would be offered by the actual contacts if they were "appropriately" distributed in the cylinder. To investigate this, an idealized model would have contacts distributed as shown in Fig. 3(b).

EXPERIMENT

To obtain data relevant to these two approaches, an electrolytic tank was set up, consisting of a closed cylinder 300 mm long, 68 mm bore, made of Perspex, with a flanged joint at its mid point, where a perforated sheet of Melinex was trapped. The model of Fig. 3(a) was represented by having one hole in the Melinex sheet, which could be placed axially or eccentrically. The model of Fig. 3(b) was represented by having patterns of many holes, generally seven holes, one central and the others on pitch radius r_p. Alternating current of mains frequency and 40 V was supplied to electrodes placed at the ends of the cylinder, but in view of the voltage drops reported by Sander [11] at electrodes, the measurements were based on voltages at probes along the length of the cylinder, observed by a bridge circuit and instruments having high impedance.

Details of construction of the apparatus, experimental procedure, results and processing of data are given in [5].

RESULTS

Figures 4(a) and 4(b) summarize the results of the tests

Results for single holes on the axis, with (r_h/r_t) ranging from 0·0263 to 0·564 were compared with the prediction of Clausius [7] quoted above. In each case the experimental value of (l_c/r_t) was between 2 and 5 per cent higher than the calculated value, but the latter makes no allowance for the thickness ($l_t = 50 \times 10^{-6}$ m) of the Melinex sheet. If an approximate allowance is made for that, by adding $l_t(r_t/r_h)^2$ to l_c, then agreement is closer, and is indicated by the points marked C on Fig. 4(a).

As shown in Fig. 4(a), for the wide range of cases considered, the effect of displacing a single hole a distance d from the axis is to increase the resistance (l_c/r_t) by an amount almost independent of the radius r_h of the hole, giving

$$\frac{l_c}{r_t} = \frac{\pi}{2}\frac{r_t}{r_h}\,\psi(r_h/r_t) + 3\left(\frac{d}{r_t}\right)^{2\cdot 5}$$

at least for $0{\cdot}0263 < (r_h/r_t) < 0{\cdot}564$ and $0 < (d/r_t) < (0{\cdot}85 - r_h/r_t)$. This rather surprising result failed to emerge from other reports of such experiments, because there the results were plotted as fractional changes in (l_c/r_t) or equivalently.

As shown in Fig. 4(b), the effect of placing seven holes in smaller groups is also to increase the resistance by an amount nearly independent of the radius r_h of the holes, at least for $0{\cdot}0104 < (r_h/r_t) < 0{\cdot}058$. The resistance was found to be at its lowest when in or near the "appropriate" distribution of Fig. 2. This lowest value was taken to represent microconstriction as discussed above (i.e. $g = r_t$ here). This value was compared with prediction based on [7], applied to a single hole of radius r_h in a cylinder of radius $(r_t/\sqrt{7})$. In each case the experimental value was between 5 and 10 per cent higher than the calculated value, but (as with single

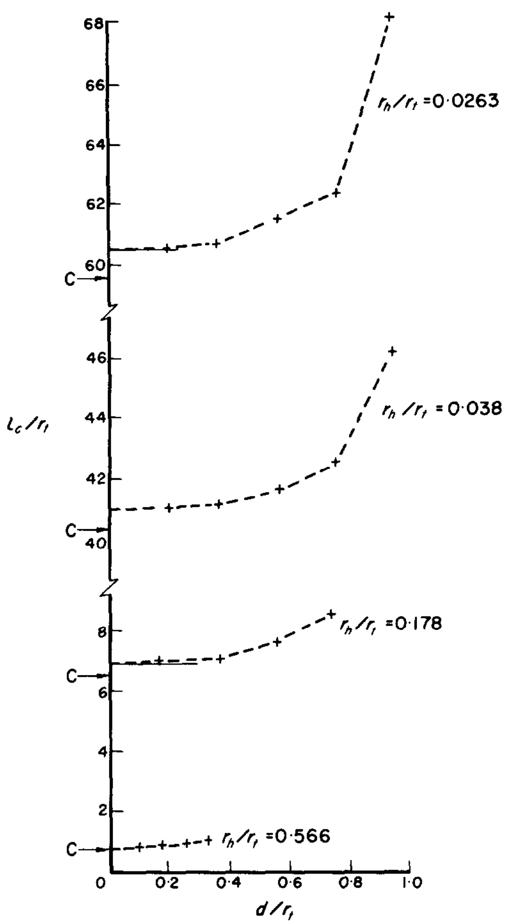


FIG. 4(a)

holes) it was necessary to make an allowance for l_p and agreement was then improved, as shown by the points marked C on Fig. 4(b). At smaller values of r_p, the resistance was greater, and the increase was expressed as a Holm radius, g, of the equivalent single contact. Details are given in [5]. For fairly compact arrangements of seven (or nineteen) holes, corresponding to values of g less than about 0·5 r_p there was found to be good agreement with the value of g predicted by Greenwood's rule of thumb for contact between infinite bodies [10]. Where g was larger than 0·5 r_t the effect of macroconstriction was small, but a linear interpolation could be used if required [5]. For values of r_p exceeding that for "appropriate" distribution, the resistance increased again, as the contact approached the outer boundary of the cylinder. The experiment was not suited to accurate investigation of those cases, which in any case are of less interest for thermal contact conductance.

It is hoped these results may be of value in establishing means of estimating thermal contact conductance for "inappropriately" spaced contacts.

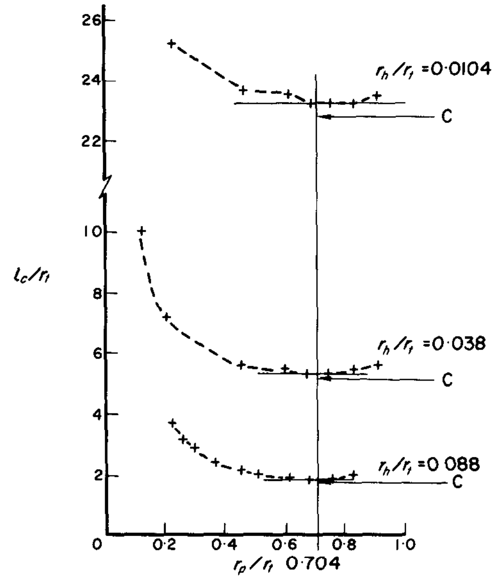


FIG. 4(b)

FIG. 4. Results of experiments. (a) single holes on or off axis, (b) seven holes, symmetrically placed. Points marked C are predicted by calculation.

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TRANSIENT EFFECTS IN MASS TRANSFER TO CIRCULATING DROPS

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NOMENCLATURE

B, dimensionless group defined by equation (14a);
b, Chao's dimensionless group defined by equation (14b);
$\mathscr{D}_{Ac}, \mathscr{D}_{Ad}$, effective pseudobinary diffusivity of solute A in the continuous and dispersed phases respectively;
j_A, radial component of the mass flux of solute A relative to the mass-average velocity;
K_d, k_d, instantaneous surface-mean mass-transfer coefficients defined by equations (4) and (5);
m, slope of equilibrium distribution isotherm, $d\rho_{Ad}/d\rho_{Ac}$;
R, sphere radius;
S, surface area;
t, time;
V_∞, approach velocity of continuous phase relative to sphere;
x, $\cos\theta$.

Greek symbols
θ, angular co-ordinate in direction of interfacial fluid motion;
μ, viscosity;
ρ, mass density;
ρ_A, mass concentration of solute A;
τ, dimensionless time defined by equation (8).

Subscripts
A, solute under consideration;
c, continuous phase;
d, dispersed phase;
s, separation point;
0, at interface;
∞, at large distance from interface.

In a recent article by Ruckenstein [8], time-dependent solutions were obtained for the species continuity equation as applied to the boundary-layer of a circulating spherical drop. These solutions were based on the steady velocity profiles as obtained from the creeping flow and potential flow approximations of the Navier–Stokes equation. In applying the surface-stretch model [1] to similar situations, it has come to our attention that the expressions obtained by Ruckenstein are applicable only to limited situations and consequently must be used with great care. In particular, equation (37), Ref. [8], for the case in which the droplet phase resistance is controlling, does not give a useful description of the mass-transfer process except over a very restricted range of Péclet numbers. Also, equations (45) and (46), Ref. [8], are based on an unrealistic velocity profile in the interfacial region for moderate Reynolds numbers.

THE SURFACE-STRETCH MODEL

An equivalent to equation (34), Ref. [8], may be obtained by using the generalized surface-stretch modification of the penetration theory [1] as applied to circulating drops. Application of the surface-stretch model gives the flux through the surface for an arbitrary surface element